

Full length article

Full-property modulation of light beam based on the unified link between amplitude, phase and polarization

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ABSTRACT

Is it possible to modulate the inherent properties of a light beam, namely amplitude, phase and polarization, simultaneously by merely its phase? Here, we solve this scientific problem by unifying all these three properties using phase vectorization and phase version of Malus's law. Full-property spatial light modulator is therefore developed based on the unification of these three properties, which enables pixel-level polarization, amplitude and phase manipulation of light beam in a real-time dynamic way. This work not only implies that the amplitude, phase and polarization of a light beam are interconnected, but also offers a reliable answer on how to modulate these three properties of a light beam simultaneously, which will deepen our understanding about the behavior of light beam, and facilitate extensive developments in optics and relate fields.

1. Introduction

Is it possible to modulate the inherent properties of a light beam, namely amplitude, phase and polarization, simultaneously, by merely its phase? The key to solve this scientific problem relies on the unification of these three attributes of a light beam. As we know, amplitude, phase and polarization are three properties of light beam. The former two are the scalar properties, while the latter refers to the electric oscillation, and is a vector property. Throughout the development of optics, the relationships between these three properties are the basis of manipulating light beam. Therefore, in the past decades, every breakthrough regarding the establishment of fundamental links between amplitude, phase and polarization has not only deepened our understanding about the behavior of light beam, but also given rise to wide-range of applications, including optical communications [1,2], advanced lasers [3–5], optical storage [6], optical displays [7,8,9], and optical imaging [10–12].

Generally, there are three familiar relationships between amplitude, phase and polarization in classical optics, namely, the mutual link between phase and amplitude, the polarization-to-amplitude link and the polarization-to-phase link. Specifically, the mutual link between phase and amplitude can be established by the wavefront technique [13–17], which serves as the cornerstone of scalar optics; the polarization-to-

amplitude link demonstrated by the renowned Malus's law implies that modulating the polarization of a light beam can be turned into amplitude adjustment with the aid of a polarizer; the Pancharatnam-Berry (PB) phase in 1987 demonstrates the polarization-to-phase link [18]. Based on this principle, one can convert a right-hand circularly polarized beam into a left-hand circularly polarized one along with an additional phase and vice versa. These above three fundamental links imply that vector polarization can link to the scalar phase and amplitude, but not the opposite. In our previous work, we solve this scientific problem using the principle of phase vectorization [19]. Accordingly, the phase-to-polarization link is established in classical optics, and the scalar phase of a light beam satisfied the wave function can link with its vector polarization directly. Even so, we can only assert that only two of amplitude, phase and polarization are interconnected. That is, these three properties of the light beam are relatively independent, and all the above four fundamental links are, to some extent, viewed as four independent links. Can we connect these four fundamental links so that all these three properties, namely phase, amplitude and polarization, of a light beam unify as a whole?

Here, we demonstrate the unification of amplitude, phase and polarization of light beam using the principle of phase vectorization and phase version of Malus's law. Taking m -order vector vortex beam (VVB) as example. Based on these two principles, three kinds of phases, named

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Phase A, B, C, are linked to the amplitude, phase and polarization of VVB, respectively. That is, all these three properties of VVB can be controlled merely by its scalar phase. Full-property spatial light modulator (SLM) is therefore developed based on the unification of amplitude, phase and polarization, which enables pixel-level polarization, amplitude and phase manipulation of light beams in a real-time dynamic way. The work reports the invention of Full-property SLM that offers a solid answer on how to modulate the phase, amplitude and polarization of a light beam simultaneously using only its phase property. More importantly, this property unification of light beam implies that amplitude, phase and polarization of light beam satisfied the wave function are interconnected.

2. Results

2.1. Main scientific goal

Although there are four fundamental relationships between the inherent amplitude, phase and polarization of a light beam, these three properties of light beam are considered to be independent from each other because only every two properties are interconnected [19]. Here, our main scientific goal is to unify these three properties using merely the scalar phase of light beam, as shown in Fig. 1. It should be emphasized that three properties of light beam can only link with three kinds of phases. Therefore, we should find out three kinds of phases that are responsible for adjusting amplitude, phase and polarization of light beam, respectively. More importantly, this property unification requires that these three kinds of phase are not affect each other. Otherwise, full property modulation of light beam cannot be achieved. For simplicity, we name these three kinds of phases as Phase A, B and C, which link to the polarization, amplitude and phase of light beam, respectively.

2.2. Phase a-to-polarization link

As already demonstrated in our previous work, phase vectorization is capable of vectorizing the scalar phase of light beam into its vector polarization, thereby establishing the fundamental relation between phase and polarization [19]. It should be emphasized that the phase and polarization are both two inherent properties of a light beam. To realize this phase-to-polarization link, two critical conditions must be satisfied: one is that the light beam must possess an inherently different polarization response from the left and right circularly polarization modes; another is that the undesired circularly polarization mode must be eliminated without affecting the desired one. The first condition implies that not all of light beams are suitable for the establishment of phase-to-polarization link. Because the phase is a scalar property, while the polarization is a vector property. The second condition indicates how to extract the desired polarization mode from a light beam.

According to both conditions, we take m -order VVB as example to vectorize its scalar phase into its vector polarization. Therefore, one can

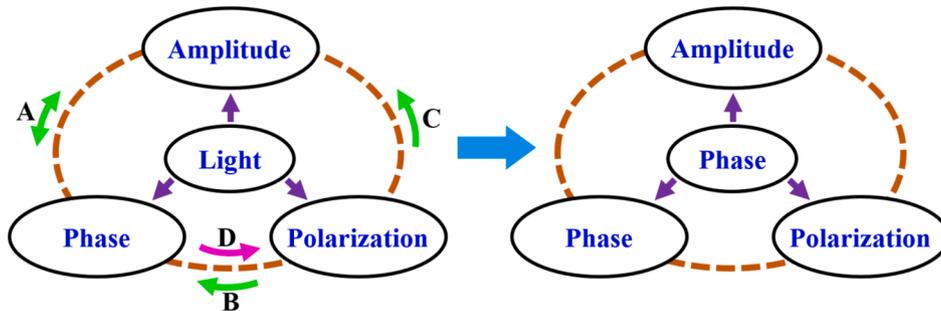


Fig. 1. Main scientific goal of our work. Here, there are four fundamental relationships between amplitude, phase and polarization of light: arrow A denotes the mutual relationship between amplitude and phase; arrow B indicates the polarization-to-phase link; and arrow C denotes the polarization-to-amplitude link. Arrow D represents the phase-to-polarization link [19]. Our main scientific goal is to unify amplitude, phase and polarization of a light beam using merely its scalar phase.

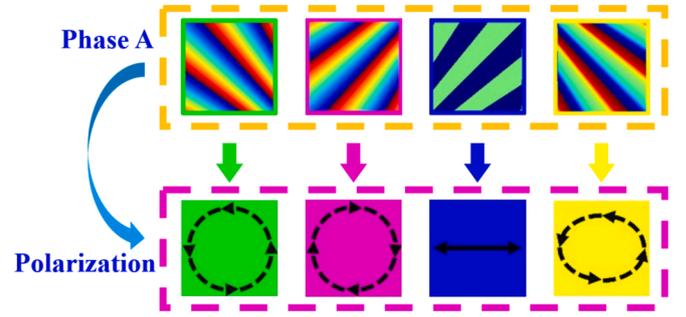


Fig. 2. Physical essence of phase vectorization. Phase vectorization presents a one-to-one correspondence between the scalar phase and vector polarization of a light beam, thereby transferring four specific phases, namely vortex phases, binary phase and a combination of both above, into left and right circular, linear, and elliptical polarization, respectively. Note that the arrow indicates the polarization direction.

simply obtain a one-to-one correspondence between the phase and polarization of m -order VVB. That is, the phase in Eq. (1) can be vectorized into the vector polarization in Eq. (2) [19].

$$\phi_A = \text{Phase}[\cos\varphi_0 \exp[i(m\varphi - \beta)] + \sin\varphi_0 \exp[-i(m\varphi - \beta)]] \quad (1)$$

$$\mathbf{E} = \cos\varphi_0 \exp(-i\beta)|\mathbf{L}\rangle + \sin\varphi_0 \exp(i\beta)|\mathbf{R}\rangle \quad (2)$$

Here, the parameter φ_0 in Eq. (1) is a weight factor that adjusts the proportion of left and right circular polarization mode $|\mathbf{L}\rangle$ and $|\mathbf{R}\rangle$. As shown in Fig. 1, the physical essence of phase vectorization is to transfer the polarization change into four specific phases: binary phase $\phi = \text{Phase}[\cos(m\varphi - \beta)]$ for linear polarization, vortex phases $\phi = \pm(m\varphi - \beta)$ for left and right circular polarization, and a combination of both for elliptical polarization. Therefore, we call the phase in Eq. (1) as Phase A, which is responsible for the polarization modulation of light beam.

2.3. Phase b-to-amplitude link

The renowned Malus's law demonstrates that modulating the polarization of a light beam can be turned into amplitude adjustment with the aid of a polarizer, thereby establishing the polarization-to-amplitude link. As shown in Supplementary Fig. 1, after passing through a polarizer, the light intensity of incident linearly polarized light beam turns into $I_o = |\mathbf{E}_0|^2 \cos^2\omega$. That is, the loss of light intensity can be obtained by $I_e = 1 - I_o = |\mathbf{E}_0|^2 \sin^2\omega$. Here, \mathbf{E}_0 is the amplitude of incident light beam and ω is the angle between the polarization direction of incident light beam and polarizer. As the polarizer rotates, both parts of light intensities I_o and I_e transform each other by ω . Therefore, it is reasonable to consider that I_o and I_e are two complementary intensity modes within a light beam that are intertwined with each other during propagating in

free space. The polarizer is utilized to separate them from each other and extract I_o . In this way, one can obtain a one-to-one correspondence relationship between polarization and amplitude of light beam.

The Malus's law implies an important physical idea that involves how to extract the inherent intensity mode I_o from a light beam. Thus, to extend this polarization-to-amplitude link into its phase version, the inherent intensity mode I_o must be extracted using the phase of light beam. As discussed above, I_o and I_e can be considered as two complementary intensity modes. During propagating in free space, both scalar modes are intertwined with each other so that the entire light beam remains stable. Although I_o cannot be extracted directly, we still can achieve an indirect scalar mode extraction of a light beam. Specifically, based on the sample optical system of phase vectorization in Fig. 5, I_o and I_e can be separated spatially in the focal region of OL₁ using optical pen [17], where ω can be adjusted by the phase of incident light beam. After passing through a pinhole, I_e is eliminated, and only I_o is retained. In this way, the phase-to-polarization link is established, thereby linking the phase B in Eq. (3) to the amplitude in Eq. (4). Detail derivation can be found in Supplementary Note 1.

$$\phi_B = \text{Phase} \left(\sum_{j=1}^N (Amp_{aj} + Amp_{bj}) \times \text{PF}(1, f_j, \eta_j, 0, 0) \right) \quad (3)$$

$$I_o = |E_0|^2 \cos^2 \omega \quad (4)$$

where

$$Amp_{aj} = \text{PF}(1, -f_j, \eta_j, 0, \omega) + \text{PF}(1, f_j, \eta_j, 0, -\omega + 0.5\pi) \quad (5)$$

$$Amp_b = \text{PF}(1, -f_j, \eta_j, 0, -\omega) + \text{PF}(1, f_j, \eta_j, 0, \omega + 1.5\pi) \quad (6)$$

Here, N indicates the number of complementary intensity modes pairs, namely I_o and I_e , in the focal region of OL₁. $\text{PF}(s_j, f_j, \eta_j, z_j, \delta_j)$ denotes the optical pen [17]. $(f_j, \eta_j, z_j = 0)$ indicate the position of j -th focus in the cylindrical coordinate system; $s_j = 1$ and δ_j are weight factors that can be used to adjust the amplitude and phase of the j -th focus, respectively.

By comparing with Malus's law, the phase B-to-amplitude link indicates an indirect amplitude mode extraction I_o of a light beam. The pinhole in Fig. 5 acts as a polarizer of Malus's law, which is responsible to eliminate the undesired amplitude mode I_e . For this reason, we call this fundamental link as phase version of Malus's law, namely phase Malus's law. As shown in Fig. 3, Phase Malus's law converts a series of phase into the amplitude of light beam. Every particular value of amplitude is correspondent with one particular kind of phase, the distribution of which is determined by the parameter N . For instance, the phase distributions for $N = 1$ and $N = 2$ are distinct. However, both configurations effectively modulate the amplitude of the light beam accordingly. Based on the phase Malus's law, the amplitude change of light beam is no longer dependent on the polarization direction of polarizer, but the phase of incident light beam. Therefore, one can simply achieve pixel amplitude modulation of light beam using pixel phase modulation device, such as phase-only SLM.

2.4. Unification of amplitude, phase and polarization

So far, we have established the phase A-to-polarization link and the phase B-to-amplitude link by the principle of phase vectorization and phase Malus's law. In principle, both fundamental links are all realized based on one physical idea, namely inherent mode extraction of a light beam. Phase vectorization implies a vector polarization mode extraction, while phase Malus's law denotes a scalar amplitude mode extraction. Unlike the phase vectorization, phase Malus's law is valid for every

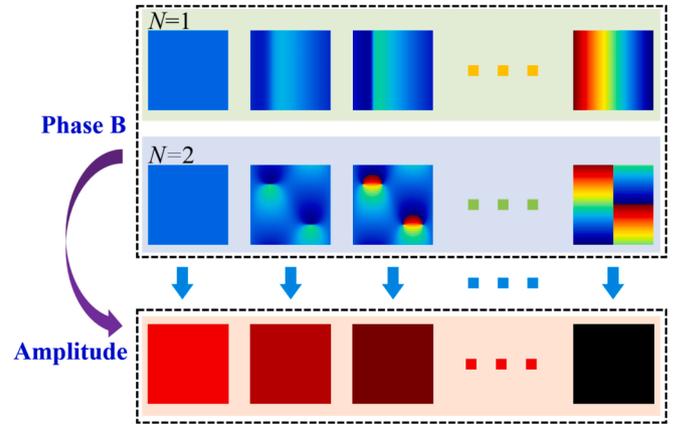


Fig. 3. Physical essence of phase Malus's law. The phase version of Malus's law, namely phase Malus's law, implies a one-to-one correspondence between the phase and amplitude of a light beam. Therefore, every particular value of amplitude is correspondent with one particular kind of phase, the distribution of which is determined by the parameter N . Here, $N = 1$ and $N = 2$ represent two distinct phases that correspond to the amplitude of the light beam.

light beam in physics, including the vector light beam using for phase vectorization. That is, Phase A and B do not affect each other because they represent different properties of light beam, namely the polarization and amplitude, respectively. As shown in Fig. 4, we therefore unify the amplitude, phase and polarization of a light beam by merely its scalar phase, which can be expressed as

$$\phi = \phi_A + \phi_B + \phi_C \quad (7)$$

Here, Phase C indicates other phases, which is responsible for the pure phase modulation. Although ϕ_A , ϕ_B , ϕ_C in Eq. (7) are overlapped together, their transmittances are divided into three parts, namely $\text{exp}i\phi_A$, $\text{exp}i\phi_B$ and $\text{exp}i\phi_C$, which are responsible for the modulation of polarization, amplitude and phase, respectively. Therefore, based on this new fundamental link, one can control all these three properties simultaneously. It should be emphasized that the light beam mentioned in this paper is the solution of wave function.

2.5. Full-property SLM

Based on the unification of amplitude, phase and polarization in Fig. 4, we establish a Full-property SLM. Full-property SLM shares a same optical system with polarized-SLM, which can also be simplified to a filter system in Fig. 5 [19]. In the entire optical system of Full-property SLM, a collimated incident x linearly polarized beam with a wavelength

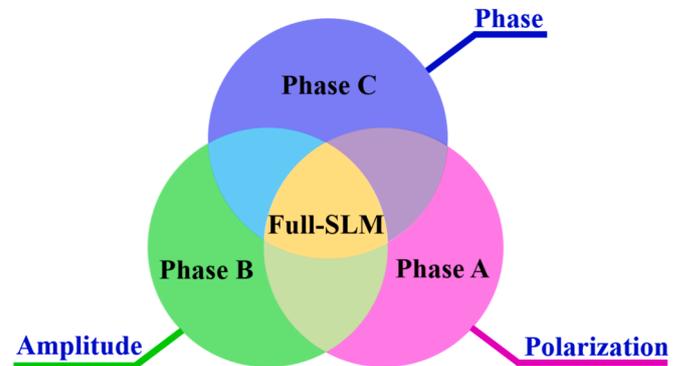


Fig. 4. Unification of amplitude, phase and polarization. Based on the property unification of light beam, three kinds of phases, namely Phase A, B, and C, are corresponding with the polarization, amplitude and phase of light beam, respectively.

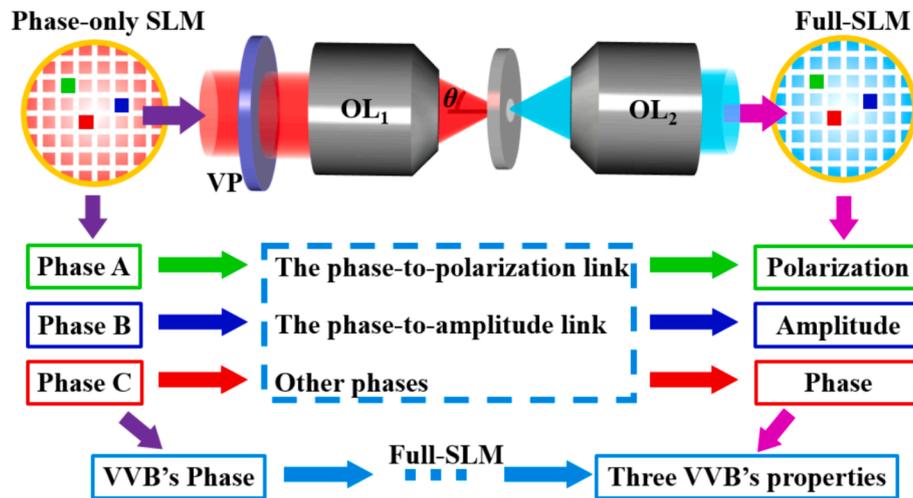


Fig. 5. Schematic of full-property SLM. A collimated incident x linearly polarized beam is converted into a $m = 30$ order VVB by a vortex polarizer (VP) after modulating by a phase-only SLM. Based on the property unification, the amplitude, polarization and phase of light beam can be adjusted simultaneously using full-property SLM.

of 633 nm propagating along the optical axis is converted into a $m = 30$ order VVB by a vortex polarizer (VP) after modulating by a phase-only SLM. The VP can be easily manufactured using the Q-plate technique [20,21]. When focusing by the first objective lens OL_1 , the modulated VVB coded by the phase A, B and C are divided into desired and undesired modes, respectively. Generally, phase A is corresponding to the desired vector polarization modes $\exp(-i\beta)|L\rangle$, $\exp(i\beta)|R\rangle$ and the undesired vector polarization modes $\exp[i(2m\varphi - \beta)]|R\rangle$, $\exp[-i(2m\varphi - \beta)]|L\rangle$, respectively. Phase B is related to the desired scalar amplitude mode I_o and the undesired scalar amplitude mode I_e , respectively. Because all desired vector and scalar modes are in the geometric focal position of OL_1 , while their counterpart undesired ones are located at the position far away from the desired modes, one can simply obtain the desired modes using a pinhole with a radius of 400 μm . That is, the desired polarized modes in the focal region of OL_1 have one-to-one correspondence with the phase A, B and C. When reconstructed

by the second objective lens OL_2 , the amplitude, phase and polarization output from Full-property SLM link with the VVB phase directly as well. In this way, the amplitude, polarization and phase of VVB are unified by merely its scalar phase. Here, the numerical apertures (NA) of both OL_1 and OL_2 are 0.01.

2.6. Pixel-level modulation using full-property SLM

Based on the above property unification of light beam, one can simply adjust its three properties merely by its scalar phase. Generally, Phase A, B and C can be pixelized by the phase-only SLM in Fig. 5. The pixelate phase enables pixelate polarization, thereby permitting individual adjustment of all these three properties in each pixel. In Fig. 6, we present three theoretical examples of performing pixel-level full-property modulation of $m = 30$ order VVB using Full-property SLM. The wavefront distribution of $m = 30$ order VVB is shown in Fig. 6 (a). In the

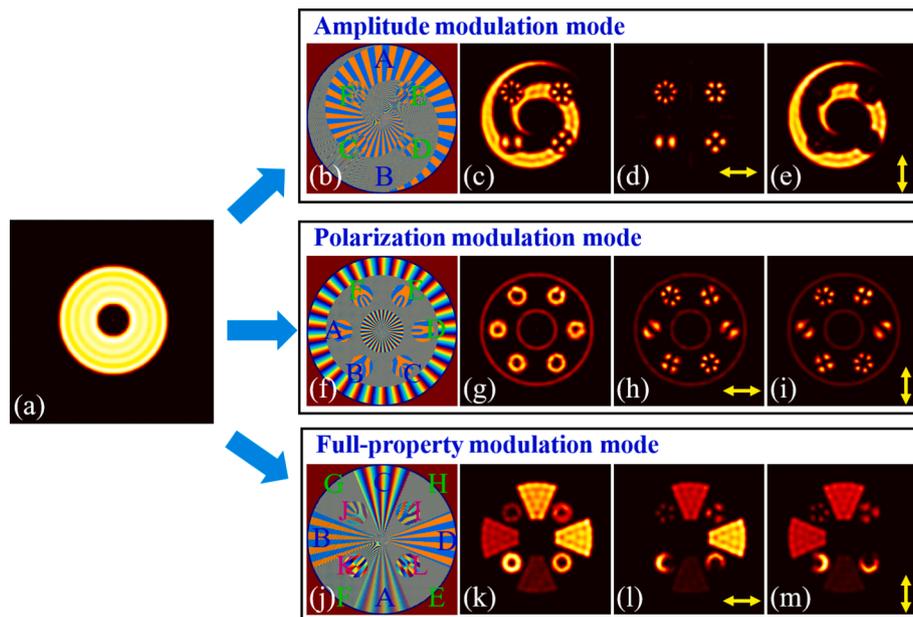


Fig. 6. Theoretical result of pixel-level full-property modulation using Full-property SLM. Here, (a) wavefront light intensity of m -order VVB. After modulating by the phases (b, f, j), one can obtain three examples of pixelate modulation of light beam: (c) amplitude modulation; (g) polarization modulation; (j) amplitude, polarization and phase modulation. (d, e), (h, i) and (l, m) are their corresponding light intensities distribution after passing through a polarizer indicated by the yellow arrow.

following simulations, we demonstrate how to modulate the amplitude, phase and polarization of VVB in Fig. 6 (a) merely by its scalar phase. Detail theoretical principle is presented Supplementary Note 2.

2.6.1. Amplitude modulation of incident m -order VVB

Fig. 6 (b-d) present a theoretical result of amplitude modulation of incident m -order VVB. The VVB phases possess two kinds of zones, namely zones A, B and zones C, D, E, F. The first kind, namely zones A, B, are x linearly polarized, while the second kind, namely zones C, D, E, F, are y linearly polarized. Both polarizations are easily realized by the phase A, which is responsible for polarization modulation in Eq. (1). After passing through a polarizer indicated by the yellow arrow, only one kind of zones are retained, as shown in Fig. 6 (c, d). In term of amplitude modulation, each zone possesses different amplitude, which can be manipulated by phase B. Specially, the amplitude of zones A and B, can be expressed as

$$I_g = |\mathbf{E}_0|^2 \cos^2 \phi_{ZAB} \quad (8)$$

while that of zones C, D, E, and F are

$$I_h = |\mathbf{E}_0|^2 \cos^2 \phi_{ZCDEF} \quad (9)$$

Here,

$$\phi_{ZAB} = \begin{cases} 0; & \text{zoneA} \quad \text{Phase}[\cos(\varphi + k\sin\theta/NA)] = -1 \\ 0.5\pi; & \text{zoneB} \quad \text{Phase}[\cos(\varphi + k\sin\theta/NA)] = 1 \end{cases}$$

$$\phi_n = \cos n\varphi$$

$n = 1, 2, 3,$ and 4 represent the circular zones C, D, E, and F, respectively.

2.6.2. Polarization modulation of incident m -order VVB

Fig. 6 (b-d) mainly verify the amplitude modulation of Full-property SLM. By comparison, we provide a theoretical result of polarization modulation in Fig. 6 (f-i). Based on the full property unification of m -order VVB, the Full-property SLM can transform m -order VVB in Fig. 6 (a) into the vector beam in Fig. 6 (g) by coding the phase in Fig. 6(f). In the transverse plane of light beam in Fig. 6 (g), there are eight different polarization states. The edges of entire light beam in Fig. 6 (a) indicated by inner and outer ring are left and right circularly polarized, respectively, which correspond to Phase A with $\phi = \pm(m\varphi - \beta)$ [see Eq. (1)], respectively. As shown in Fig. 6 (g), the amplitude within the light beam are zero except 6 circular zones. These zones are also named as zone A, B, C, D, E, and F, respectively. Light beams in these zones possesses the polarization state of VVB with different order, which are correspondent with the phase A in Fig. 6 (f). Here, the polarization of zones A, B, C can be expressed as

$$P_n = \begin{bmatrix} \cos n\varphi + \beta \\ \sin n\varphi + \beta \end{bmatrix} \quad (10)$$

where $\beta = 0.25\pi$; $n = 1, 2, 3$ represent zones A, B, C, respectively, and $n = -1, -2, -3$ represent zones D, E, F, respectively. Because of the plus-minus sign of n , after passing through the polarizer indicated by the yellow arrows, the petals with $2n$ numbers are rotated along with the polarizer reversely, as shown in Fig. 6 (h, i).

2.6.3. Full-property modulation of incident m -order VVB

To some extent, Fig. 6 (b, g) demonstrate the amplitude and polarization modulation of Full-property SLM. Theoretically, it is not a convincing answer for the scientific question: how to modulate the three properties of a light beam, namely phase, amplitude and polarization, merely by its scalar phase. The reason is that phase A and B are spatially divided in the transverse plane of m -order VVB. That is, one can only achieve those properties manipulation in different zone instead of an identical zone. It should be emphasized that full-property modulation of light beam requires to manipulate all these three properties, namely

amplitude, phase and polarization, of light beam in an identical pixel by its scalar phase, as shown in Fig. 7. Therefore, we verify the full-property modulation of light beam using Full-property SLM in Fig. 6 (j-m). As shown in Fig. 6 (j), the phase of m -order VVB possesses eight fatal-like zones, namely zones (A-D) and (E-H). The light intensities of zones (E-H) are adjusted to be zero, except the four small circular zones, namely zones (I-L). As shown in Fig. 6 (k), one can not only adjust the light intensity of one zone, but also modulate the polarization of an identical zone by overlapping Phase A with Phase B. Specifically, as shown in Fig. 6 (l, m), the polarization states of zones (A-D) are x linear polarization, left circular polarization, y linear polarization and right circular polarization, respectively, while their corresponding light intensities are 0.25, 0.5, 0.75, 1, respectively.

As discussed above, the polarization, amplitude, and phase are corresponding to the Phase A, B and C, respectively. When superposing all these three kinds of phases, one can obtain different light beam with different polarization, different phase and even different amplitude in the same zone of light beam, thereby providing a full property modulation in the zones (I-L), see Fig. 6 (k). The light beams in zones (I-L) are propagable vortex beams carrying natural non-integer orbital angular momentum. Their peculiar polarizations can be expressed as [22]

$$\mathbf{E}_{mj} = \exp[i(l + 0.5)\varphi] \begin{bmatrix} \cos[(m + 0.5)\varphi + \beta] \\ \sin[(m + 0.5)\varphi + \beta] \end{bmatrix} \quad (11)$$

where $l = -1, m = -2, \beta = 0.5\pi$ for zone I; $l = 0, m = -3, \beta = 0.5\pi$ for zone J; $l = 1, m = 0, \beta = 0.5\pi$ for zone K; $l = 2, m = -1, \beta = 0.25\pi$ for zone L. According to Eq. (11), these vector beams not only possess complicate polarization state but also carry a natural optical vortex $\exp[i(l + 0.5)\varphi]$. After passing through the polarizers indicated by the yellow arrow in Fig. 6 (l, m), petals with number $2m + 1$ are rotated along with the polarizer, which further demonstrates the polarization in each zone. Moreover, the amplitudes of zones (I, J, L, K) in Fig. 6 (k) are adjusted to 0.25, 0.5, 0.75, 1, respectively. Note that the polarization distribution in zones (I-L) can be found in our previous work [22].

2.6.4. Distinction between the pixel of a phase-only SLM and that of a full-property SLM

Finally, it is essential to clarify the distinction between the pixel of a phase-only SLM and that of a full-property SLM in Fig. 5. Each pixel of the phase-only SLM in Fig. 5 can only adjust the scalar phase of a light beam, representing a single degree of freedom. In contrast, the pixel of a full-property SLM is more intricate. For example, as shown in Fig. 2, four specific phases can be converted into four distinct polarization states based on the fundamental relationship between phase and polarization. Thus, the pixel of full-property SLM contains a unique phase distribution determined by the polarization state of the incident m -order VVB, enabling control over the polarization of the VVB. Similarly, the pixel for amplitude modulation in the full-property SLM, as depicted in Fig. 3, is distinct from that of a commercial phase-only SLM as well. These pixels function as super-pixels, capable of transforming specific phases into different amplitudes. Consequently, when overlapping Phase A, Phase B and Phase C, a full-property SLM can simultaneously control the polarization, phase, and amplitude of a light beam.

Additionally, the resolution of a full-property SLM is inherently lower than that of a phase-only SLM, primarily due to the filtering process depicted in Fig. 5, which the desired mode must pass through. Generally, a smaller pixel size in a phase-only SLM correlates with a higher resolution in a full-property SLM. This relationship stems from the fundamental hardware limitations of phase-only SLM. Historically, early SLM featured pixel sizes exceeding $100 \mu\text{m}$. In contrast, contemporary SLM has significantly reduced pixel sizes. As SLM technology continues to advance, we anticipate a further decrease in pixel size, which will consequently enhance the resolution of full-property SLMs. However, it is important to note that this hardware limitation does not compromise the validity of full-property SLM.



Fig. 7. Schematic of full property modulation. Based on the property unification of light beam, the amplitude, polarization and phase of light beam can be modulated simultaneously in an identical pixel using merely its scalar phase, namely Phase A, B and C.

3. Discussion

3.1. Physical idea of full-property unification of light beam

In the following, we would like to discuss the physical idea of full property unification of a light beam. In our works, we are aim to unify the three properties, namely phase, amplitude and polarization, of a light beam. Therefore, the phase-to-polarization link and the phase-to-amplitude link are established in classical optics by proposing the principle of phase vectorization and phase Malus's law. Frankly speaking, there is one physical idea that is the basis of both fundamental links, namely extraction inherent vector and scalar modes from a light beam, as shown in Fig. 8.

3.1.1. Vector mode extraction of a light beam

Generally, in our optical text book, we are told that every new light beam can normally be created by superposing two orthogonal polarized beams with different phase in an interferometric optical system, which is also indicated by Jones matrix theory. Based on this physical idea, one can only obtain one particular new light beam when the parameters of two orthogonal polarized beams are determined. That is, this conventional principle implies a one-to-one correspondence in the framework of generating a new light beam. No doubt, mathematically, one can

always obtain $1 + 1 = 2$, instead of other values. However, extraction inherent modes from a light beam manifests an entirely inverse process of the above generalized principle. Similar to the mathematical form, we can not only obtain $2 = 1 + 1$, but also $2 = 3 - 1$ and $2 = 5 - 3$ as well. Therefore, this inverse process provides plenty of possibilities to extract a desired light beam. This physical thought conveys an entirely new principle of polarization modulation, thereby making it possible to unify the amplitude, phase and polarization of light beam merely by its scalar phase.

3.1.2. Scalar mode extraction of a light beam

Similarly, the conventional Malus's law implies a physical idea that involve how to extract the inherent intensity modes from a light beam. Specifically, one can obtain $I_o = |\mathbf{E}_o|^2 \cos^2 \omega$ of a light beam by $I_e = |\mathbf{E}_o|^2 \sin^2 \omega$ using a polarizer. Both scalar modes I_o and I_e are two complementary intensity modes that are intertwined with each other during propagating in free space. The polarizer is utilized to separate them from each other and extract I_o . Derived from the identical physical idea of the conventional Malus's law, the Phase B-to-Amplitude link is a direct process of scalar mode extraction, but an indirect one using merely the phase of a light beam, see Supplementary Note 1. For this reason, we call it the phase version of Malus's law, namely phase Malus's law. Although there are plenty of previous works that can also modulate the amplitude of light beam with its phase [13,14], the phase Malus's law conveys a different physical idea, namely mode extraction of a light beam. Note that we are not concentrated on the technical details of above techniques, but their different physical idea.

3.1.3. Technical advance of full-property modulation

According to the above discussions, the unified link between amplitude, phase, and polarization is established based on a single physical concept: the inherent mode extraction of a light beam. Extracting inherent modes from a light beam is essentially the inverse process of the generalized superposition principle of two light beams. Every light mode output from a full-property SLM can be considered as a particular mode within the incident m -order VVB. Therefore, without overlapping two different light beams, arbitrary light beam can be realized directly by adjusting the phase of the m -order VVB. In this way, a phase-only SLM can be simply converted into a full-property SLM. Compared to the superposition of two light beams, full-property SLM can be simplified into a simple filter system as shown in Fig. 5. When creating a new light beam, it no longer requires complicated algorithm, high interferometrically precise alignment, or complex optical path design, which represents one of the most significant technical advantages of the full-property SLM.

Although pixel-level full-property modulation of a light beam can be achieved using a full-property SLM, the quality of the light beam can be affected by the pinhole. In principle, both desired and undesired modes exist in the focal region of OL_1 . When propagating in free space, the undesired mode is eliminated by the pinhole shown in Fig. 5, and only the desired mode is reconstructed by OL_2 . Therefore, the pinhole must



Fig. 8. Schematic of inherent mode extraction of a light beam. Both phase vectorization and phase Malus's law rely on one physical idea, namely extraction inherent arbitrary modes from a light beam, including vector and scalar mode of light beam.

be placed between the desired and undesired modes. Generally, it is better to increase the size of the pinhole to provide sufficient space for the modulation of the desired mode. A larger space for the desired mode enables the creation of more complex light modes using the full-property SLM. Fortunately, the size of the pinhole can be increased with the increment of the order of the incident VVB, as reported in Ref. [19].

3.2. Conclusion

In conclusion, we have theoretically demonstrated that all the three properties of light beam, namely amplitude, phase and polarization, can be unified by its scalar phase using the principle of phase vectorization and phase Malus's law. Based on this unified link, a Full-property spatial light modulator (SLM) is developed, which enables pixel-level polarization, amplitude and phase manipulation of light beam in a real-time dynamic way. This work presents the unification of the three properties of light beam, which may not only deepen our understanding about the fundamental relationship between the properties of light beam, but also find valuable applications in optical communications, optical imaging, and optical display, among others.

CRediT authorship contribution statement

Xiaoyu Weng: Writing – review & editing, Methodology, Investigation, Conceptualization. **Fangrui Lin:** Writing – review & editing, Methodology, Investigation. **Zhiyang Xia:** Methodology, Investigation. **Yu Chen:** Writing – review & editing, Visualization. **Liwei Liu:** Writing – review & editing, Supervision. **Jun He:** Writing – review & editing, Validation. **Changrui Liao:** Validation, Investigation. **Yiping Wang:** Validation, Supervision. **Junle Qu:** Validation, Supervision, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.optlastec.2025.113059>.

Data availability

Data will be made available on request.

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